

Decoherence of two qubits in a non-Markovian squeezed reservoir

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The decoherence of two initially entangled qubits in a squeezed vacuum cavity has been investigated exactly. The results show that, first, in principle, the disentanglement time decreases with the increasing of squeeze parameter r , due to the augmenting of average photon number of every mode in squeezed vacuum cavity. Second, there are entanglement revivals after complete disentanglement for the case of even parity initial Bell state, while there are entanglement decrease and revival before complete disentanglement for the case of odd parity initial Bell state. The results are quite different from that of the case for qubits in a vacuum cavity.

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I. INTRODUCTION

In recent years, the entanglement dynamics of qubits, coupled with environment, has attracted much attention. Many works have been devoted to the phenomenon termed as “entanglement sudden death” (ESD) [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. It is shown that spontaneous disentanglement, for two-level atom model, may take only a finite-time to be completed, while local decoherence (the normal single-atom transverse and longitudinal decay) takes an infinite time [3]. And, for a non-Markovian reservoir of initially vacuum cavity, there is revival phenomenon, which allows the two-qubit entanglement to reappear after a dark period of time, during which the concurrence is zero [12]. In general, the characteristic of the environment plays an important role in the evolution of multi-particle entanglement. Up to now, many typical environments have been investigated, such as, vacuum, squeezed vacuum, multimode vacuum cavity, single mode cavity and so on.

Recently, cavity systems with very strong couplings have been discussed [16]. Generally, in atom-field cavity systems, this ratio is typically of the order $10^{-7} \sim 10^{-6}$. However, the ratio may become order of magnitudes larger in solid state systems, and the full Hamiltonian, including the virtual processes (counter-rotating terms), must be considered [17]. In this paper, we will focus on the decoherence of two qubits strongly coupled with a non-markovian squeezed vacuum reservoir by the method in Ref. [18]. In section.2, the reduced non-perturbative non-Markovian quantum master equation of atom is derived and its exact solution is obtained. In section.3, the decoherence of two initially entangled atoms, coupled with two cavities separately, has been discussed. The conclusion is given in section.4.

II. MODEL AND EXACT SOLUTION

A. Hamiltonian and non-perturbative master equation

Now we restrict our attention to two noninteracting two-level atoms A and B coupled individually to the cavity field. To this aim, we first consider the Hamiltonian of the subsystem of a single qubit coupled to its reservoir as

$$H = H_a + H_r + H_{ar} \quad (1)$$

where

$$H_a = \omega_0 \frac{\sigma_z}{2} \quad (2)$$

$$H_r = \sum_k \omega_k a_k^\dagger a_k \quad (3)$$

$$H_{ar} = (\sigma_+ + \sigma_-) \sum_k g_k (a_k^\dagger + a_k) \quad (4)$$

where ω_0 is the atomic transition frequency between the ground state $|0\rangle$ and excited state $|1\rangle$. $\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$, $\sigma_+ = |1\rangle\langle 0|$ and $\sigma_- = |0\rangle\langle 1|$ are pseudo-spin operators of atom. The index k labels the field modes of the reservoir with frequency ω_k , a_k^\dagger and a_k are the modes' creation and annihilation operators, and g_k is the frequency-dependent coupling constant between the transition $|1\rangle - |0\rangle$ and the field mode k .

The reduced non-perturbative non-Markovian quantum master equation of atom could be obtained by path integral [18]

$$\frac{\partial}{\partial t} \rho_a = -i \mathcal{L}_a \rho_a - \int_0^t ds \langle \mathcal{L}_{ar} e^{-i \mathcal{L}_0(t-s)} \mathcal{L}_{ar} e^{-i \mathcal{L}_0(s-t)} \rangle_r \rho_a \quad (5)$$

where \mathcal{L}_0 , \mathcal{L}_a and \mathcal{L}_{ar} are Liouvillian operators defined as

$$\mathcal{L}_0 \rho \equiv [H_a + H_r, \rho]$$

$$\mathcal{L}_a \rho \equiv [H_a, \rho]$$

$$\mathcal{L}_{ar} \rho \equiv [H_{ar}, \rho]$$

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and $\langle \dots \rangle_r$ stands for partial trace of the reservoir.

Then, we assume the reservoir is initially in squeezed vacuum state[19]

$$\rho_r = \prod_k S_k |0_k\rangle \langle 0_k| S_k^\dagger \quad (6)$$

$$S_k = \exp(re^{-i\theta} a_{k_c+k} a_{k_c-k} - re^{i\theta} a_{k_c+k}^\dagger a_{k_c-k}^\dagger) \quad (7)$$

r, θ are squeeze parameters, $|0_k\rangle$ is the vacuum state of mode k . The central frequency of squeezing device is $\omega_0 = ck_c$, which corresponds to multimode squeezed vacuum state with the central frequency equal to the cavity resonance frequency and atom transition frequency. And the spectral density of the reservoir is in Lorentzian form[12, 20]

$$\begin{aligned} J(\omega) &= \sum_k g_k^2 \{ \delta(\omega - \omega_k) + \delta(\omega + \omega_k) \} \\ &= \frac{1}{2\pi} \frac{\lambda \gamma^2}{(\omega - \omega_0)^2 + \gamma^2} \end{aligned} \quad (8)$$

where γ represents the width of the spectral distribution of the reservoir modes and is related to the correlation time of the noise induced by the reservoir, $\tau_r = 1/\gamma$. The parameter λ is related to the subsystem-reservoir coupling strength. There are two correlation functions in this model[21]

$$\alpha_1 = \int_{-\infty}^{\infty} J(\omega) e^{-i(\omega - \omega_0)t} = \frac{\gamma \lambda}{2} e^{-\gamma t} \quad (9)$$

$$\alpha_2 = \int_{-\infty}^{\infty} J(\omega) e^{i(\omega + \omega_0)t} = \frac{\gamma \lambda}{2} e^{(-\gamma + i2\omega_0)t} \quad (10)$$

α_1 comes from the rotating-wave interaction and α_2 from the counter-rotating wave interaction.

So, the non-perturbative master equation of the subsystem could be derived from Eq.(5)

$$\begin{aligned} \frac{\partial}{\partial t} \rho_a &= -\Gamma \rho_a + [\varepsilon_0 J_0 + \varepsilon_+ J_+ + \varepsilon_- J_-] \rho_a \\ &\quad + [v_0 K_0 + v_+ K_+ + v_- K_-] \rho_a \end{aligned} \quad (11)$$

where J_0, J_+, J_-, K_0, K_+ and K_- are superoperators defined as

$$\begin{aligned} J_0 \rho_a &\equiv \left[\frac{\sigma_z}{4}, \rho_a \right] \\ J_+ \rho_a &\equiv \sigma_+ \rho_a \sigma_+ \\ J_- \rho_a &\equiv \sigma_- \rho_a \sigma_- \\ K_0 \rho_a &\equiv (\sigma_+ \sigma_- \rho_a + \rho_a \sigma_+ \sigma_- - \rho_a) / 2 \\ K_+ \rho_a &\equiv \sigma_+ \rho_a \sigma_- \\ K_- \rho_a &\equiv \sigma_- \rho_a \sigma_+ \end{aligned}$$

and

$$\begin{aligned} \Gamma &= 2M^R f + 2(M^R \alpha^R + M^I \alpha^I) + (2N + 1)(f + \alpha^R) \\ \varepsilon_0 &= -i2[\omega_0 + 2(M^I \alpha^R - M^I f - M^R \alpha^R) - (2N + 1)\alpha^R] \\ \varepsilon_+ &= 2Mf + 2M^* \alpha + (2N + 1)(f + \alpha) \\ \varepsilon_- &= 2M\alpha^* + 2M^* f + (2N + 1)(f + \alpha^*) \end{aligned}$$

$$\begin{aligned} v_0 &= 2(\alpha^R - f) \\ v_+ &= 2[M^R f + M^R \alpha^R + M^I \alpha^I + Nf + (N + 1)\alpha^R] \\ v_- &= 2[M^R f + M^R \alpha^R + M^I \alpha^I + (N + 1)f + N\alpha^R] \end{aligned}$$

$$\alpha = \frac{\lambda \gamma (1 - e^{-(\gamma + i2\omega_0)t})}{2(\gamma + i2\omega_0)}$$

$$f = \frac{\lambda}{2} [1 - \exp(-\gamma t)]$$

$N = \sinh^2 r$, $M = -e^{i\theta} \sinh r \cosh r$. $\alpha^R, \alpha^I, \alpha^*$ and M^R, M^I, M^* are real part, image part and conjugate of α and M , respectively. Here, N is the average photon number of every mode in squeezed vacuum cavity. And M, M^* represent the phase-dependent correlation between different modes as $\langle b_k b_{k'} \rangle$ and $\langle b_k^\dagger b_{k'}^\dagger \rangle$, respectively[19].

B. Exact solution of master equation

The time evolution of density operator in Eq.(11) could be obtained with algebraic approach in Ref.[22] because the superoperators herein satisfy $SU(2)$ Lie algebraic communication relations, i.e.

$$\begin{aligned} [J_-, J_+] \rho_a &= -2J_0 \rho_a \\ [J_0, J_\pm] \rho_a &= \pm J_\pm \rho_a \\ [K_-, K_+] \rho_a &= -2K_0 \rho_a \\ [K_0, K_\pm] \rho_a &= \pm K_\pm \rho_a \\ [K_i, J_j] &= 0 \end{aligned} \quad (12)$$

where $i, j = 0, \pm$. By directly integrating Eq.(11), the formal solution is obtained as[23]

$$\begin{aligned} \rho_a(t) &= e^{-\Gamma_k \hat{T}} e^{\int_0^t dt (\varepsilon_0 J_0 + \varepsilon_+ J_+ + \varepsilon_- J_-)} \\ &\quad \times \hat{T} e^{\int_0^t dt (\nu_0 K_0 + \nu_+ K_+ + \nu_- K_-)} \rho_a(0) \end{aligned} \quad (13)$$

where \hat{T} is time ordering operator and

$$\begin{aligned} \Gamma_k &= (2M^R + 2N + 1)(F + \tilde{\alpha}^R) + 2M^I \tilde{\alpha}^I \\ \tilde{\alpha} &= \int_0^t \alpha dt = \frac{\lambda \gamma (t - \frac{1 - e^{-(\gamma + i2\omega_0)t}}{\gamma + i2\omega_0})}{2(\gamma + i2\omega_0)} \equiv \tilde{\alpha}^R + i\tilde{\alpha}^I \\ \tilde{\alpha}^* &= \tilde{\alpha}^R - i\tilde{\alpha}^I \\ F(t) &= \lambda \{ t - [1 - \exp(-\gamma t)] / \gamma \} / 2 \end{aligned}$$

where $\tilde{\alpha}^R, \tilde{\alpha}^I$ and $\tilde{\alpha}^*$ are real part, image part and conjugate of $\tilde{\alpha}$, respectively.

The exponential functions of superoperators in Eq.(13) could be disentangled in form[23]

$$\hat{T} e^{\int_0^t dt (\varepsilon_0 J_0 + \varepsilon_+ J_+ + \varepsilon_- J_-)} = e^{j_+ J_+} e^{j_0 J_0} e^{j_- J_-} \quad (14)$$

$$\hat{T} e^{\int_0^t dt (\nu_0 K_0 + \nu_+ K_+ + \nu_- K_-)} = e^{k_+ K_+} e^{k_0 K_0} e^{k_- K_-} \quad (15)$$

where j_+, j_0, j_- and k_+, k_0, k_- satisfy the following differential equation

$$\dot{X}_+ = \mu_+ - \mu_- X_+^2 + \mu_0 X_+ \quad (16)$$

$$\dot{X}_0 = \mu_0 - 2\mu_- X_+ \quad (17)$$

$$\dot{X}_- = \mu_- \exp(X_0) \quad (18)$$

$\mu = \varepsilon$ for $X = j$ and $\mu = \nu$ for $X = k$.

Using the results above, the exact solution of the master equation Eq.(11) is obtained

$$\rho_a(t) = e^{-\Gamma_k} \tilde{\rho}(t) \quad (19)$$

$$\tilde{\rho}(t) = \begin{pmatrix} l\rho_a^{11}(0) + m\rho_a^{00}(0) & x\rho_a^{10}(0) + y\rho_a^{01}(0) \\ q\rho_a^{01}(0) + r\rho_a^{10}(0) & n\rho_a^{00}(0) + p\rho_a^{11}(0) \end{pmatrix} \quad (20)$$

$$l = e^{k_0/2} + e^{-k_0/2} k_+ k_-, \quad m = e^{-k_0/2} k_+ \quad (21)$$

$$n = e^{-k_0/2}, \quad p = e^{-k_0/2} k_- \quad (22)$$

$$q = e^{-j_0/2}, \quad r = e^{-j_0/2} j_- \quad (23)$$

$$x = e^{j_0/2} + e^{-j_0/2} j_+ j_-, \quad y = e^{-j_0/2} j_+ \quad (24)$$

C. Concurrence

In order to investigate the entanglement dynamics of the bipartite system, we use Wootters concurrence[24]. For simplicity, we assume that the two subsystems have the same parameters. The concurrence of the whole system could be obtained[6]

$$C_\xi = \max\{0, c_1, c_2\}, \quad (\xi = \Phi, \Psi) \quad (25)$$

$$c_1 = 2e^{-2\Gamma_k} (\sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}})$$

$$c_2 = 2e^{-2\Gamma_k} (\sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}})$$

corresponding to the initial states of $|\Phi\rangle = \beta|01\rangle + \eta|10\rangle$ and $|\Psi\rangle = \beta|00\rangle + \eta|11\rangle$, respectively. Where β is real and $0 < \beta < 1$, $\eta = |\eta|e^{i\varphi}$ and $\beta^2 + |\eta|^2 = 1$. For maximum entanglement Bell state, β is equal to $\sqrt{2}/2$. The reduced joint density matrix of the two atoms, in the standard product basis $\mathfrak{B} = \{|1\rangle \equiv |11\rangle, |2\rangle \equiv |10\rangle, |3\rangle \equiv |01\rangle, |4\rangle \equiv |00\rangle\}$, could be obtained by the method in ref.[12]

$$\rho^{AB} = e^{-2\Gamma_k} \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix} \quad (26)$$

here the diagonal elements are

$$\begin{aligned} \rho_{11} &= l^2 \rho_{11}(0) + lm \rho_{22}(0) + ml \rho_{33}(0) + m^2 \rho_{44}(0) \\ \rho_{22} &= lp \rho_{11}(0) + lm \rho_{22}(0) + mp \rho_{33}(0) + mn \rho_{44}(0) \\ \rho_{33} &= lp \rho_{11}(0) + pm \rho_{22}(0) + nl \rho_{33}(0) + nm \rho_{44}(0) \\ \rho_{44} &= p^2 \rho_{11}(0) + pn \rho_{22}(0) + np \rho_{33}(0) + n^2 \rho_{44}(0) \end{aligned}$$

and the nondiagonal elements are

$$\begin{aligned} \rho_{14} &= x^2 \rho_{14}(0) + xy \rho_{23}(0) + yx \rho_{32}(0) + y^2 \rho_{41}(0) \\ \rho_{23} &= xr \rho_{14}(0) + xq \rho_{23}(0) + yr \rho_{32}(0) + yq \rho_{41}(0) \\ \rho_{32} &= rx \rho_{14}(0) + ry \rho_{23}(0) + qx \rho_{32}(0) + qy \rho_{41}(0) \\ \rho_{41} &= r^2 \rho_{14}(0) + rq \rho_{23}(0) + qr \rho_{32}(0) + q^2 \rho_{41}(0) \end{aligned}$$

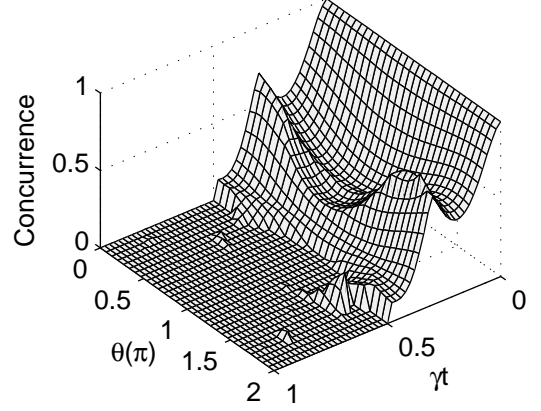


FIG. 1: Concurrence C_Φ as a function of γt and θ with $\lambda = 10\gamma$, $\omega_0 = 10\gamma$, $\beta = \sqrt{2}/2$ and $r = 0.2$.

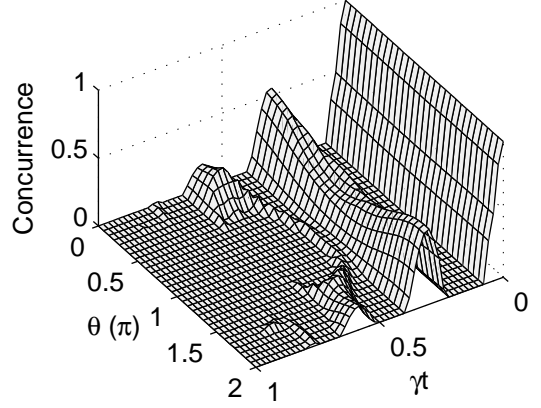


FIG. 2: Concurrence C_Ψ as a function of γt and β^2 with $\lambda = 10\gamma$, $\omega_0 = 10\gamma$, $\beta = \sqrt{2}/2$ and $r = 0.2$.

III. NUMERICAL RESULTS AND DISCUSSION

In order to study the effects of non-Markovian squeezed reservoir on the decoherence, we assume that λ is principally equal to 10γ in Eq.(8), which could be realized in a high-Q cavity[12].

First, we focus on the effects of squeeze parameters on the decoherence of two qubits with initial maximum Bell entanglement states $|\Phi\rangle$ and $|\Psi\rangle$, respectively.

(A) For $\omega_0 = 10\gamma$ and $\beta = \sqrt{2}/2$, Fig.1 and Fig.2 show that, for the case of initial maximum Bell state $|\Phi\rangle$, the concurrence first decreases to a certain value and then revives before it vanishes, while that periodically vanishes and revives with a damping of its revival amplitude for the case of initial maximum Bell state $|\Psi\rangle$. It also reveals that the amplitude and the duration time of entanglement revival are different for the case of $\pi/2 \leq \theta \leq 3\pi/2$ and for the case of $0 \leq \theta \leq \pi/2$ and $3\pi/2 \leq \theta \leq 2\pi$.

(B) For $\omega_0 = 10\gamma$ and $\beta = \sqrt{2}/2$, Fig.3 and Fig.4 show

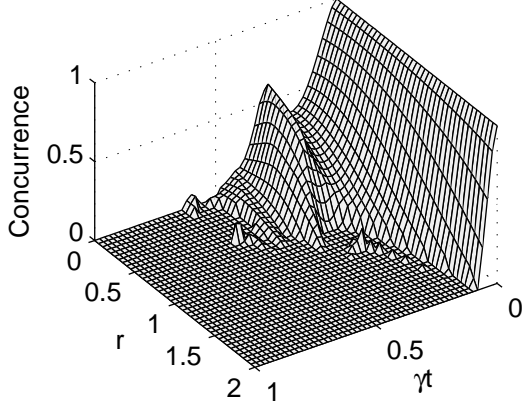


FIG. 3: Concurrence C_Φ as a function of γt and r with $\lambda = 10\gamma$, $\omega_0 = 10\gamma$, $\beta = \sqrt{2}/2$ and $\theta = \pi/4$.

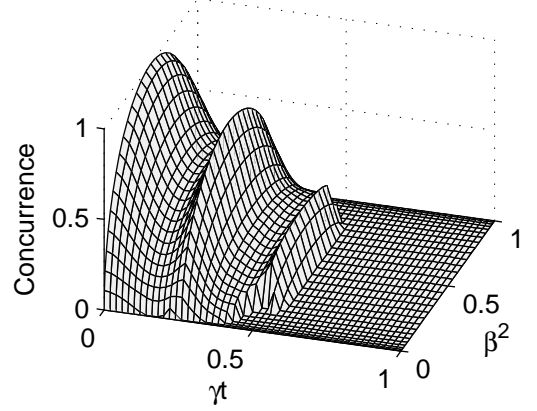


FIG. 5: Concurrence C_Φ as a function of γt and β^2 with $\lambda = 10\gamma$, $\omega_0 = 12\gamma$, $\theta = \pi/4$ and $r = 0.2$.

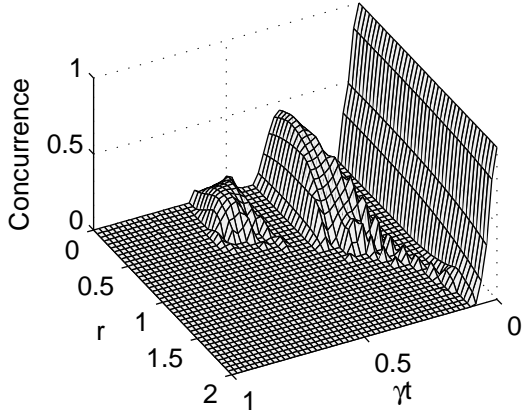


FIG. 4: Concurrence C_Ψ as a function of γt and r with $\lambda = 10\gamma$, $\omega_0 = 10\gamma$, $\beta = \sqrt{2}/2$ and $\theta = \pi/4$.

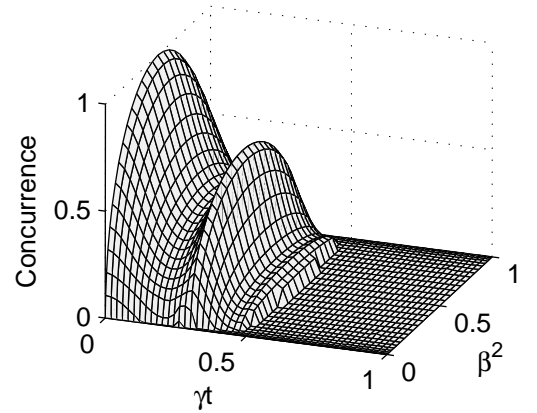


FIG. 6: Concurrence C_Φ as a function of γt and β^2 with $\lambda = 10\gamma$, $\omega_0 = 10\gamma$, $\theta = \pi/4$ and $r = 0.2$.

that, for $r \leq 1$, the concurrence first decreases to a certain value and then revives before it vanishes for the case of initial maximum Bell state $|\Phi\rangle$, while that periodically vanishes and revives with damping amplitude for the case of initial maximum Bell state $|\Psi\rangle$. And the concurrence decreases monotonically and vanishes permanently after a short time for $r > 1$. The results exhibit that the disentanglement time decreases with the increasing of squeeze parameter r for fixed θ , due to the increasing of average photon number of every mode in squeezed vacuum cavity.

The above results also reveal that the decoherence behavior of concurrence is sensitive to squeeze parameters and insensitive to initial state.

Then, for fixed squeeze parameters $\theta = \pi/4$ and $r = 0.2$, the decoherence for different initial states was discussed.

(A) The decoherence behavior of C_Ψ is discussed as follows.

(1) From Fig.5, Fig.6 and Fig.7, we could find that

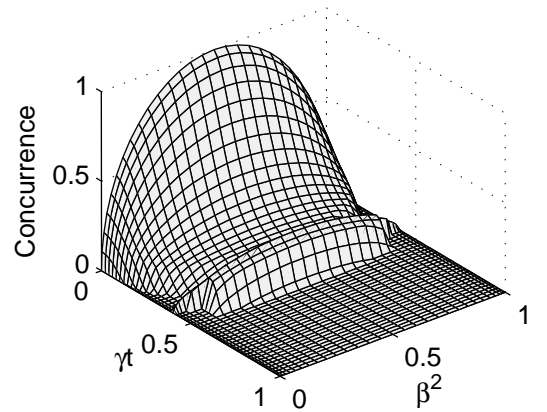


FIG. 7: Concurrence C_Φ as a function of γt and β^2 with $\lambda = 10\gamma$, $\omega_0 = 6.5\gamma$, $\theta = \pi/4$ and $r = 0.2$.

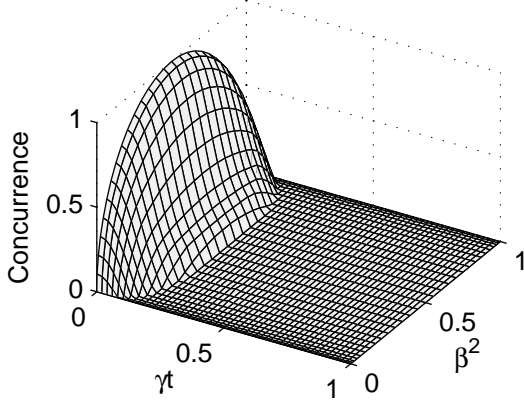


FIG. 8: Concurrence C_Φ as a function of γt and β^2 with $\lambda = 10\gamma$, $\omega_0 = 3\gamma$, $\theta = \pi/4$ and $r = 0.2$.

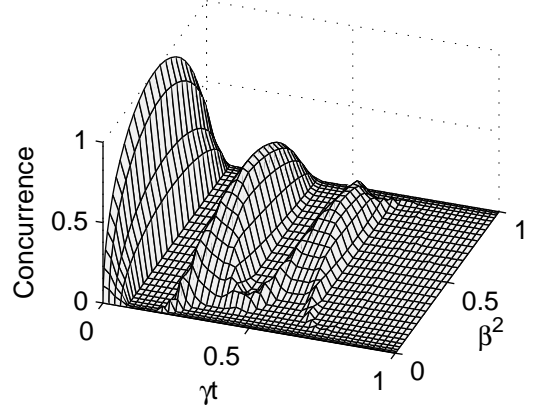


FIG. 10: Concurrence C_Ψ as a function of γt and β^2 with $\lambda = 10\gamma$, $\omega_0 = 10\gamma$, $\theta = \pi/4$ and $r = 0.2$.

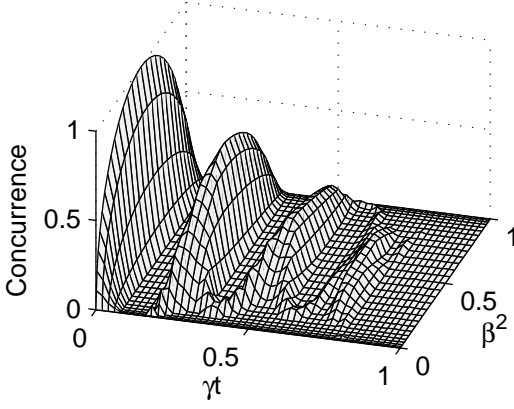


FIG. 9: Concurrence C_Ψ as a function of γt and β^2 with $\lambda = 10\gamma$, $\omega_0 = 12\gamma$, $\theta = \pi/4$ and $r = 0.2$.

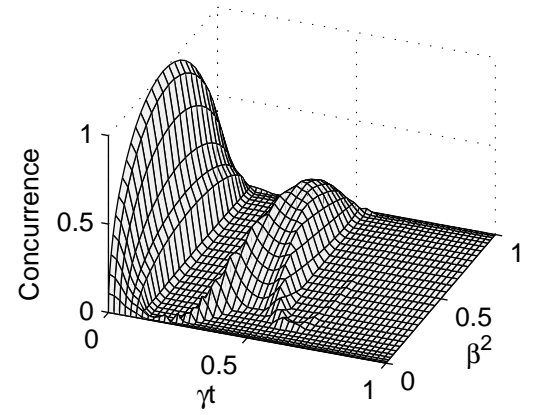


FIG. 11: Concurrence C_Ψ as a function of γt and β^2 with $\lambda = 10\gamma$, $\omega_0 = 6.5\gamma$, $\theta = \pi/4$ and $r = 0.2$.

the concurrence C_Φ will decrease and revive before it vanishes permanently. With the ratio of the coupling strength to the atomic frequency increasing, the times of entanglement revival decrease because of the increasing of the average value of correlation function α_2 , corresponding to the counter-rotating wave terms.

(2) For $\omega_0 = 3\gamma$ and $\lambda = 20\gamma$, Fig.8 exhibits that the concurrence C_Φ decreases monotonically and vanishes permanently in a short time even in a non-Markovian squeezed reservoir, resulted from the strong interaction between atom and the non-Markovian squeezed reservoir.

(B) The decoherence behavior of C_Ψ is discussed as follows.

(1) From Fig.9, Fig.10 and Fig.11, we find that the concurrence periodically vanishes and revives with a damping of its revival amplitude. With the ratio of the coupling strength to the atomic frequency increasing, the times of entanglement revival decrease. There are entanglement revivals after a period of time of disen-

tanglement, which are different from that of the case of C_Φ .

(2) For $\omega_0 = 2\gamma$ and $\lambda = 20\gamma$, the evolution dynamics of concurrence C_Ψ is almost the same as that in Fig.8. Unlike the two cases above, the evolution behavior of concurrence C_Ψ becomes symmetric because the strong coupling of atom with non-Markovian reservoir and the effect of counter-rotating wave interaction.

The above results reveal that, in principal, the decoherence of C_Φ is symmetrical with β^2 because of the symmetry of initial state $|\Phi\rangle$, while that is unsymmetrical with β^2 because the initial state $|\Psi\rangle$ is unsymmetrical with β^2 . However, the strong coupling and the counter-rotating wave interaction could make the decoherence of C_Ψ become symmetrical with β^2 . With the enhancing of coupling strength, the memory effect of the counter-rotating wave terms becomes dominant because the the average value of correlation function α_2 increases. The results are quite different from that of the case for qubits

in a vacuum cavity[12].

IV. CONCLUSION

The reduced non-perturbative quantum master equation of atom in initially squeezed vacuum cavity has been derived and its exact solution is obtained. The decoherence behaviors of two qubits with squeeze parameters and other parameters have been discussed.

The results show that the decoherence behavior of two qubits in a squeezed reservoir is dependent on the squeeze parameter, the ratio of the coupling strength to the atomic transition frequency and the ratio of the width of reservoir spectral density to the atomic transition frequency. First, in principle, the disentanglement time decreases with the increasing of squeeze parameter r , due to the increasing of average photon number

of every mode in squeezed vacuum cavity. Second, there are entanglement revivals after complete disentanglement for the case of even parity initial Bell state, while there are entanglement decrease and revival before complete disentanglement for the case of odd parity initial Bell state. Third, with the enhancing of coupling strength, the times of entanglement revival decrease due to the effect of counter-rotating wave terms. Fourth, there is entanglement revival due to the memory effect of the non-Markovian squeezed environment.

Acknowledgments

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